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MEASUREMENT UNCERTAINTIES AND INCONSISTENCIES

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INTRODUCTION

The concept of accuracy is generally understood. However, statements like "an accuracy of 1%" are common. What is really meant is 99% accuracy. This discrepancy between the number and the concept has lead to national laboratories abandoning the term by except in a qualitative, e.g. "high accuracy," context. It has been replaced by the term uncertainty. The phrase "an uncertainty of 1%" might seem equivalent to the older accuracy statement, but what does it really mean? Is 1% the maximum, average or typical variation one can expect? Often, users do not know the uncertainty of their results, and interpret any variations as inconsistencies. Two laboratories, each using different equipment or conditions, may be measuring different values because the light source actually has values that depend on the conditions. This is a real inconsistency. On the other hand, the laboratories may get different results, but each is within the uncertainty of measurement. This is pure chance.

A common question asked by people making measurements is "What is a reasonable variation in results that I can expect with my equipment?" The answer is not simple since it depends on what is being measured and the conditions of measurement. It is understandable that users link result repeatability with equipment reliability. So Optronic Laboratories created an Excel™ spreadsheet to show the optimum conditions for making NVIS radiance measurements to the user's repeatability requirements. This paper describes the basis of the spreadsheet.

The purpose of making a measurement is to get a result. It is a common misconception that there is a "right" result and that any other value involves an error in the measurement. The word "error" implies a mistake, and although mistakes are possible *(and are common causes of inconsistencies)* this is not the subject of this paper. Instead, it will be concentrating on normal variations that occur in all measurements.

The quote "There are three types of lies: lies, damned lies and statistics" is attributed to Benjamin Disraeli, a British prime minister in the 1880's who is often used to reflect the popular perception of this branch of mathematics. Statistics is often perceived negatively because it is sometimes misused to "prove" things, but in reality it can only describe what is likely or unlikely. To explain uncertainties, we need to use statistics. Hopefully, by the end of the paper you will perceive statistics as a useful tool that can save you time and enhance confidence in results.

AVERAGES AND STANDARD DEVIATIONS

The statistical quantities that apply to measurements are derived from the average and standard deviations of a set of results. To explain these terms, we can start with a random event: the toss of a coin. The coin is equally likely to fall as heads *(H)* or tails *(T)*. Now throw two coins at the same time. The four possible results can be: HH, HT, TH, TT. Since the combination of one head and one tail occurs twice, it is twice as likely as two heads or two tails. Now throw ten coins at the same time. There are 1024 *(2^{10})* possible combinations. There is still a chance of throwing ten heads or ten tails, but it would only happen once in 1024 throws on average. We could calculate the permutations and combinations to tell us how likely each result is, but we could also just throw the coins a large number of times and note how many heads occurred in each toss. If we plotted the number of occurrences for each possible number of heads, we see that although the way each of the coins fall is random, not all results are equally likely. As shown in Figure 1a, the characteristic bell-shape of a normal distribution is generated.

If we divide the number of occurrences by the total number of throws, we get the probabilities for each occurrence, as shown in Figure 1b. Probability is just a number showing likelihood between 0 *(will never happen)* and 1 *(happens every time)*. The total probability, represented by the area of the bell-shape, is always 1.

From Figure 1b, we can see for instance that the chance of throwing 5 heads and 5 tails is about 25%. This demonstrates an important concept: previous results can be used to predict the likelihood of future results.

You can see a line describing the bell-shape in Figure 1b. This is calculated from two parameters: average and standard deviation. Gauss postulated the calculation formula, and the curve is often referred to as a Gaussian distribution. By using these two parameters, we can describe the distribution without giving the graphs in Figure 1.

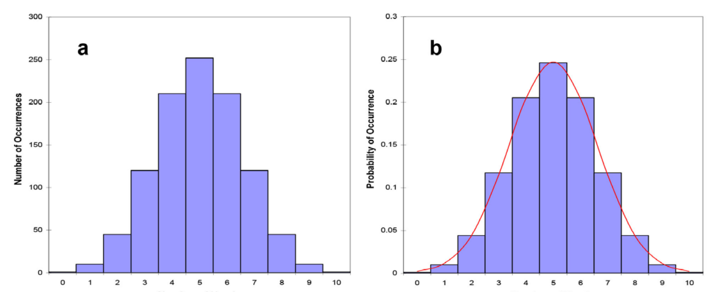

Figure 1. Results of throwing 10 coins 1024 times. a) Plot of number of occurrences vs. number of heads. b) The same plot but dividing the number of occurrences by 1024 to give probability, and a Gaussian curve fitted to the data (line).

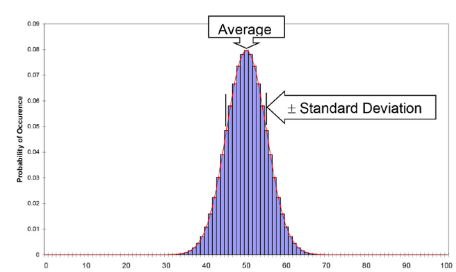

Figure 2. Probability distribution for throwing 100 coins. Showing the Gaussian curve (line) with average and standard deviation.

If we throw 100 coins, the number of possible results is very large *(2100)*. However, we still get the familiar bell-shape normal distribution, as shown in Figure 2. Also shown is the Gaussian curve with the average *(50)* and ±1 standard deviation *(±5)*. Standard deviation is calculated from the differences of individual results from the average. In Excel, the function to use for this is STDEV(). You can see that the standard deviation is a measure of the width of the bell-shaped distribution and so is a direct measure of expected variations in results.

The standard deviation also defines the confidence in the result. For instance, about 67% of results lie within ±1 standard deviation. We can therefore be 67% confident that the next throw of 100 coins will result in 50 ±5 heads *(i.e. 45 – 55)*. About 95% of results lie within ±2 standard deviations, so we can be 95% confident that a result would be 50 ±10 heads *(i.e. 40 – 60)*. So where does uncertainty fit in? Uncertainty is effectively the limits, generally expressed as percentage relative to the average, at some stated confidence level and so is related directly to the average and standard deviation.

REAL DATA

All measurements include some random variations, often called noise. Whatever it is called or how it is caused, results will have an average value and a standard deviation just like the coin toss examples. NVIS radiance measurements are unusual in that the signals are often very low at longer wavelengths and the uncertainty in results is dominated by noise. This means that there may be a substantial variability between individual measurements. In this condition, the variability in results can be calculated from a simple measurement of dark current noise. The ability to calculate the variability under different measurement conditions can be very useful, in that it can indicate whether repeat scans are necessary or even help in selection of appropriate conditions to achieve reliable results.

Before going through the calculation, we need to determine the standard deviation of the dark current. This involves making several measurements of dark current, as shown in Figure 3.

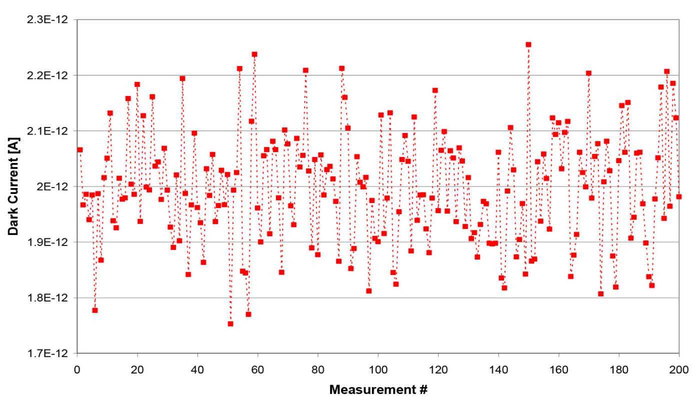

Figure 3. Measurements of dark current.

The standard deviation of data in Figure 3, given by the Excel STDEV() function, is 1 x 10^{-13}A. During a measurement, a dark current reading is taken and is then subtracted from the signals during the wavelength scan to give net signals. As the signal approaches the dark current, as is typical at longer wavelengths of an NVIS radiance measurement, the net signal depends strongly on exactly which dark value was read. As an example, let us use the data in Figure 3. If the dark current were the lowest point on Figure 3, then all the other net signals would be positive. Similarly, if the dark current were the highest, all other net signals would be negative. This would lead to offsets in the average net signals of 2.43 x 10^{-13}A and –2.59 x 10^{-13}A, respectively. This might not seem significant but will increase or decrease the NVIS radiance result. Only when the dark current value is the average will the net signal average to zero and the correct NVIS radiance be obtained.

There are two influences of dark noise on NVIS radiance results: one is from the dark current reading and the other is from noise during scans. They have the same standard deviation (σs), but do not affect results equally. The dark current is a value applied to all points in the spectrum, but noise during scans has highs and lows and can cancel out to some extent. The uncertainty in unscaled NVISa radiance results due to dark noise is given by:

$$\sigma_{NVISa} = \sqrt{1 + \frac{1}{\int_{450}^{930} G_A(\lambda)\cdot d\lambda}} * \int_{450}^{930} \sigma_s \cdot C(\lambda)\cdot G_A(\lambda)\cdot d\lambda \qquad \textbf{Equation 1}$$

where C(λ) are the instrument calibration values and $G_A(\lambda)$ are the NVISa response values. The first part of the sum is the effect of the dark current value, and the second part is the effect of averaging noise during the scan. A similar equation applies to NVISb radiance but with $G_B(\lambda)$ response values instead of $G_A(\lambda)$ values. When applying this uncertainty value, it must be scaled to the luminance to agree with scaled NVIS radiance calculations.

REDUCING UNCERTAINTIES IN RESULTS

In order to reduce noise and get closer to the average dark current level, multiple measurements can be combined. The noise reduces with the square root of the number of measurements. If we write Equation 1 in a more general way to include the number of dark readings (N_D) and number of scans (S), we get:

$$\sigma_{NVISa} = \sqrt{\frac{\left(N_D + \int_{450}^{930} G_A(\lambda)\cdot d\lambda\right)}{S \cdot N_D \cdot \int_{450}^{930} G_A(\lambda)\cdot d\lambda}} * \int_{450}^{930} \sigma_s \cdot C(\lambda)\cdot G_A(\lambda)\cdot d\lambda \qquad \textbf{Equation 2}$$

Basically, increasing either the number of dark readings or the number of scans decreases the uncertainty. Extra dark readings take much less time than extra scans however, so is generally preferred.

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of measurement error that can be encountered if calibrations are not performed properly to compensate for effects due to absorption and reflection properties of LEDs and LED holders.

The effect of the LED itself depends more on the color of the epoxy package rather than the emission spectrum. A transparent LED hardly changes the calibration, but colored epoxy packages can produce up to 10% changes to the calibration factors. Figure 5 shows the ratios of calibration factors with various combinations of LEDs and LED holders to those without an LED holder.

THE SPREADSHEET

This is readily available from Optronic Laboratories or you can create your own using Equation 2. The operation of the Optronic Laboratories version, shown in Figure 4, will be described.

Essentially, rather than just calculating an uncertainty, the spreadsheet is aimed at advising the user on suitable conditions to achieve desired results. The user provides an acceptable NVIS radiance result variation and the standard deviation of the PMT dark noise of the system used. The calibration files are pasted into the spreadsheet on a worksheet not shown in Figure 4.

The uncertainty in the scaled NVISa and NVISb radiances are calculated for each luminance condition and aperture at k=1, k=1 meaning at ±1 standard deviation. These values are then compared to the "acceptable NVIS radiance result variation" criterion, and a table of conditions *(aperture in rows, luminance in columns)* is marked "OK" to indicate conformance or "AVOID" to indicate a condition that should not be used. The user can simply select the conditions most conducive to the test being run.

If the condition that needs to be used is marked "AVOID," the user simply increases the values for number of scans or number of dark readings until that condition indicates "OK," as shown in Figure 5. This saves a lot of time that might be wasted scanning with unsuitable conditions or using trial-and-error methods.

Measurement Conditions		
PMT Noise standard deviation =	1.00E-13	A
Acceptable NVIS radiance result variation =	2.00E-11	W/(cm² sr)
Number of scans =	1	
Number of dark readings =	2	

Expected minimum NVIS radiance variation (at k=1)

	130-NVG lens									
	NVISa					NVISb				
fL	0.3	0.5	1	3	5	0.3	0.5	1	3	5
0.1	8.01E-11	3.36E-11	8.88E-12	1.97E-12	1.29E-12	7.67E-11	3.22E-11	8.50E-12	1.89E-12	1.23E-12
0.2	4.01E-11	1.68E-11	4.44E-12	9.84E-13	6.43E-13	3.84E-11	1.61E-11	4.25E-12	9.43E-13	6.16E-13
0.5	1.60E-11	6.73E-12	1.78E-12	3.94E-13	2.57E-13	1.53E-11	6.44E-12	1.70E-12	3.77E-13	2.46E-13
1	8.01E-12	3.36E-12	8.88E-13	1.97E-13	1.29E-13	7.67E-12	3.22E-12	8.50E-13	1.89E-13	1.23E-13
2	4.01E-12	1.68E-12	4.44E-13	9.84E-14	6.43E-14	3.84E-12	1.61E-12	4.25E-13	9.43E-14	6.16E-14
5	1.60E-12	6.73E-13	1.78E-13	3.94E-14	2.57E-14	1.53E-12	6.44E-13	1.70E-13	3.77E-14	2.46E-14
15	5.34E-13	2.24E-13	5.92E-14	1.31E-14	8.57E-15	5.11E-13	2.15E-13	5.67E-14	1.26E-14	8.21E-15

Suitability of conditions to giving acceptable result

	130-NVG lens									
	NVISa					NVISb				
fL	0.3	0.5	1	3	5	0.3	0.5	1	3	5
0.1	AVOID	AVOID	OK	OK	OK	AVOID	AVOID	OK	OK	OK
0.2	AVOID	OK	OK	OK	OK	AVOID	OK	OK	OK	OK
0.5	OK	OK	OK	OK	OK	OK	OK	OK	OK	OK
1	OK	OK	OK	OK	OK	OK	OK	OK	OK	OK
2	OK	OK	OK	OK	OK	OK	OK	OK	OK	OK
5	OK	OK	OK	OK	OK	OK	OK	OK	OK	OK
15	OK	OK	OK	OK	OK	OK	OK	OK	OK	OK

Figure 4. Results page of the Optronic Laboratories spreadsheet. The user enters values in the boxes under "Measurement Conditions" and other values are calculated automatically.

Measurement Conditions		
PMT Noise standard deviation =	1.00E-13	A
Acceptable NVIS radiance result variation =	2.00E-11	W/(cm² sr)
Number of scans =	2	
Number of dark readings =	20	

Suitability of conditions to giving acceptable result

	130-NVG lens									
	NVISa					NVISb				
fL	0.3	0.5	1	3	5	0.3	0.5	1	3	5
0.1	OK	OK	OK	OK	OK	OK	OK	OK	OK	OK
0.2	OK	OK	OK	OK	OK	OK	OK	OK	OK	OK
0.5	OK	OK	OK	OK	OK	OK	OK	OK	OK	OK
1	OK	OK	OK	OK	OK	OK	OK	OK	OK	OK
2	OK	OK	OK	OK	OK	OK	OK	OK	OK	OK
5	OK	OK	OK	OK	OK	OK	OK	OK	OK	OK
15	OK	OK	OK	OK	OK	OK	OK	OK	OK	OK

Figure 5. Sections of the results page, showing that changing the number of scans and dark readings can provide optimisation of conditions.

SETTING CONDITIONS

All of the previous discussions assume that the noise can be averaged to zero when the NVIS radiance is zero. This means that negative numbers must be allowed. Some users dislike seeing negative values of radiance, since such a condition cannot exist in reality. However, we are discussing measured values here, and measurements must include both positive and negative values to average to zero. Artificially replacing negative values by zero will mean the average always remains positive and gives larger NVIS radiances than is really present.

In Figure 4, the calculated uncertainties in NVIS radiances are headed "Expected minimum NVIS radiance variation." This is because the dominant source of variation is assumed to be noise. If other sources of uncertainty such as radiance changes in the display or temperature variations occur, these will increase the overall uncertainty in results.

CONCLUSIONS

Utilizing uncertainties to optimise measurement conditions shows that statistics can help users, even if they have no knowledge of statistics. It enables them to plan measurements that give reliable results, increasing their confidence. It also indicates expected variations, allowing them to distinguish between normal variations and inconsistencies. For instance, suppose you manufacture a display and your measurements indicate it passes MIL-L-85762A specifications. You send it to your customer who measures and rejects it as a failure. If the difference between the NVIS values is less than the combined uncertainties of both measurements, the disagreement is the result of chance variations and it may well pass if the customer re-measured it. If the difference exceeded the combined uncertainties, then there may well be an inconsistency in how the display is measured e.g. measuring different areas of the display.